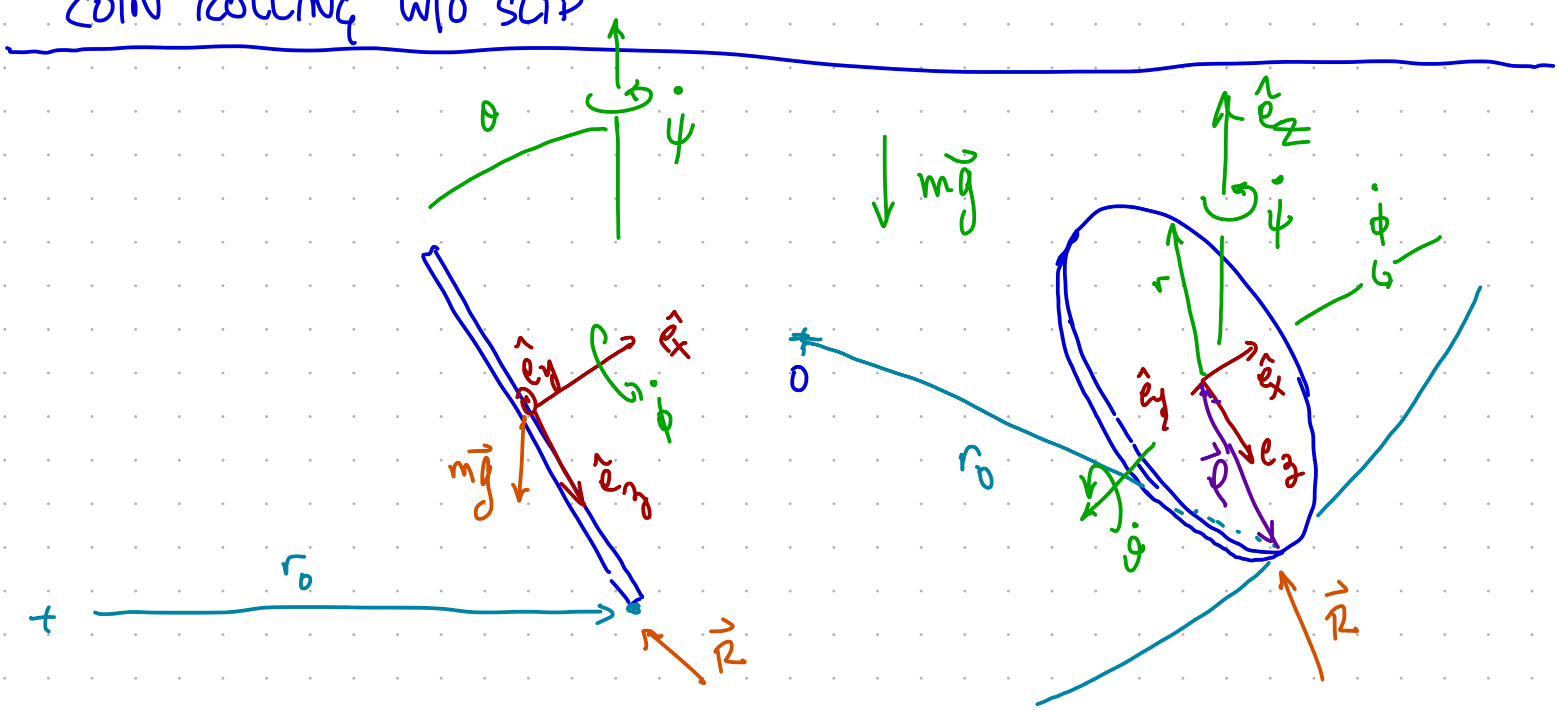
$R\dot{\psi} = L\dot{\theta} \Rightarrow \dot{\psi} = \frac{L}{R}\dot{\theta}$ — HOLONOMIC.

Hmm... $\dot{\psi} = \frac{L}{R}(\dot{\theta})\left(1 - e^{-t/\tau}\right)$ — NON-HOLONOMIC

VIRTUAL POWER FOR RIGID BODIES

$$\underbrace{\left(m\ddot{\vec{r}}_c - \vec{F}_a\right) \cdot \frac{\partial \dot{\vec{r}}_c}{\partial \dot{q}_k}}_{\text{LINEAR MOMENTUM - FORCE}} + \underbrace{\left(\dot{\vec{H}}_c - \vec{M}_{c,a}\right) \cdot \frac{\partial \vec{\omega}_c}{\partial \dot{q}_k}}_{\text{ANGULAR}} = 0$$

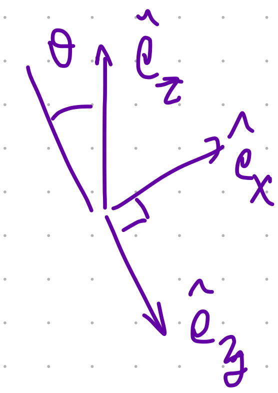
COIN ROLLING W/O SLIP



POSITION VECTOR: (from O)

$\vec{r}_c =$ —— UM, YEAH, WELL WHERE'S OUR REFERENCE?

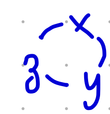
OOOH... IT'S THE CONTACT POINT...



$$\dot{\vec{r}}_c = \underbrace{\vec{\omega}}_{\dot{\theta}\hat{e}_y + \dot{\psi}\underbrace{\hat{e}_z}_{-\hat{e}_3\cos\theta + \hat{e}_x\sin\theta} + \dot{\phi}\hat{e}_x} \times \underbrace{\vec{\rho}}_{-r\hat{e}_3}$$

$$\Rightarrow \dot{\vec{r}}_c = \left[(\dot{\phi} + \dot{\psi}\sin\theta)\hat{e}_x + \dot{\theta}\hat{e}_y - \dot{\psi}\cos\theta\,\hat{e}_3\right] \times (-r\hat{e}_3)$$

$$= r(\dot{\phi} + \dot{\psi}\sin\theta)\hat{e}_y - r\dot{\theta}\hat{e}_x$$



$$\Rightarrow \ddot{\vec{r}}_c = r(\ddot{\phi} + \ddot{\psi}\sin\theta + \dot{\psi}\dot{\theta}\cos\theta)\hat{e}_y +$$

$$r(\dot{\phi} + \dot{\psi}\sin\theta)\dot{\hat{e}}_y - r\ddot{\theta}\hat{e}_x - r\dot{\theta}\dot{\hat{e}}_x$$

$$\dot{\hat{e}}_y = \underbrace{\vec{\omega}'}_{\dot{\psi}\hat{e}_z} \times \hat{e}_y \qquad \dot{\hat{e}}_x = \underbrace{\vec{\omega}''}_{\dot{\psi}\hat{e}_z + \dot{\theta}\hat{e}_y} \times \hat{e}_x \quad *$$

$$= \dot{\psi}\hat{e}_z \times \hat{e}_y$$

$$= (\dot{\psi}\sin\theta\,\hat{e}_x - \dot{\psi}\cos\theta\,\hat{e}_3) \times \hat{e}_y$$

$$= \dot{\psi}\sin\theta\,\hat{e}_3 + \dot{\psi}\cos\theta\,\hat{e}_x$$

$*$ $$\dot{\hat{e}}_x = \vec{\omega}'' \times \hat{e}_x = (\dot{\psi}\hat{e}_z + \dot{\theta}\hat{e}_y) \times \hat{e}_x = \left[(\dot{\psi}\sin\theta)\hat{e}_x + \dot{\theta}\hat{e}_y - \dot{\psi}\cos\theta\,\hat{e}_3\right] \times \hat{e}_x$$

$$-\dot{\theta}\hat{e}_3 - \dot{\psi}\cos\theta\,\hat{e}_y$$

$$\ddot{\vec{r}}_c = r(\ddot{\phi} + \ddot{\psi}\sin\theta + \dot{\psi}\dot{\theta}\cos\theta)\,\hat{e}_y + r(\dot{\phi} + \dot{\psi}\sin\theta)(\dot{\psi}\sin\theta\,\hat{e}_z + \dot{\psi}\cos\theta\,\hat{e}_x) - r\ddot{\theta}\,\hat{e}_x$$
$$+ r\dot{\theta}(\dot{\theta}\,\hat{e}_z + \dot{\psi}\cos\theta\,\hat{e}_y)$$
$$= \hat{e}_x\left[r(\dot{\phi} + \dot{\psi}\sin\theta)(\dot{\psi}\cos\theta) - r\ddot{\theta}\right] + \hat{e}_y\; r(\ddot{\phi} + \ddot{\psi}\sin\theta + \dot{\psi}\dot{\theta}\cos\theta + \dot{\theta}\dot{\psi}\cos\theta)$$
$$+ \hat{e}_z\left[r(\dot{\phi} + \dot{\psi}\sin\theta)\dot{\psi}\sin\theta + r\dot{\theta}^2\right]$$

$$\vec{F}^a = mg(-\hat{e}_Z) = mg(-\sin\theta\,\hat{e}_x + \cos\theta\,\hat{e}_z)$$

THEN we can write the rectilinear equations of motion: $(m\ddot{\vec{r}}_c - \vec{F}_a)\cdot\frac{\partial\dot{\vec{r}}_c}{\partial\dot{q}_k} = 0$

BUT we also have the angular motion $(\dot{\vec{H}}_c - \vec{M}_{c,a})\cdot\frac{\partial\vec{\omega}}{\partial\dot{q}_k} = 0$.

HEYYY... $\vec{M}_{c,a} = \vec{0}$! NOW $\vec{H}_c$ IS NEEDED; $\vec{H}_c \equiv \mathbb{I}_c\,\vec{\omega}_c$ ← $\vec{\omega}$ FROM ABOVE

MAYBE WE CAN ASSUME THAT THE PRODUCTS OF INERTIA ARE ZERO, $I_{c_{ij}} = 0$ for $i \neq j$

$$\vec{H}_c = I_{xx_c}\omega_x\hat{e}_x + I_{yy_c}\omega_y\hat{e}_y + \underbrace{I_{yy_c}}_{=I_{33c}\text{ by symmetry}}\omega_z\hat{e}_z\,,$$