Be sure to read and carefully go through Chapter 1.

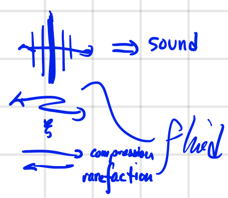
NO CLASS NEXT WEEK: HOLIDAY!

WEEK AFTER (28 JAN): MEET IN SME 320.

ACOUSTICS

FLUIDS cannot support shear. (As opposed to solids).
So, in fluids, sound is compressional ONLY.

Suppose we have a sound source in our fluid, oscillating at an amplitude of ξ.
Sound is at frequency, f, and propogates at a speed c.
Then the wavelength is $\lambda = c/f$.



The distance between adjacent compressed and rarefied regions will be $\lambda/2 \sim \lambda$
"on the order of"

thermal diffusivity: α (assuming you remember what thermal diffusivity is...)

The time required to equilibriate the temp. from the compressed to rarefied regions is $\sim \lambda^2/\alpha$.

Compare this to the time for one wavelength of sound to pass: $\frac{1}{f}$ ...

RATIO: $\left(\frac{\lambda^2}{\alpha}\right)(f) \sim$ RATIO OF THERM. EQUIL. TIME TO THE ACOUSTIC WAVE PERIOD

LOOK: $\frac{\lambda^2 f}{\alpha} = \frac{c^2}{f\alpha} = f_0/f$ if we define $f_0 \equiv c^2/\alpha$: If $f \ll f_0$, then the temperature WILL NOT EQUILIBRATE

AIR: $c \sim 10^2$ m/s and $\alpha \sim 10^{-5}$ m²/s $\Rightarrow f_0 \sim 10^{10}$ Hz ~ 10 GHz
WATER: $c \sim 10^3$ m/s and $\alpha \sim 10^{-7}$ m²/s $\Rightarrow f_0 \sim 10$ THz.

So sound is adiabatic as $f \ll f_0$ in most cases. Losses due to imperfect collisions (i.e., viscosity is different)

Sound propagation concepts

So sound is adiabatic ... and the equation of state for an adiabatic fluid is

$$p \propto \rho^{\gamma}, \quad \gamma = \begin{Bmatrix} 1.0 & \text{for water} \\ 1.4 & \text{for air} \\ 5/3 & \text{for helium} \end{Bmatrix} = c_p / c_v$$

p: pressure; ρ: density; c_p: SPECIFIC HEAT @ CONST. PRES.; c_v: SPECIFIC HEAT @ CONST. VOL.

Let's look at a compressed region of the fluid... as the sound passes through. BEFORE: width of $\frac{\lambda}{2}$

AFTER: (compressed) $\frac{\lambda}{2} - \left(\frac{\xi}{2}\right) - \left(\frac{\xi}{2}\right) = \frac{\lambda}{2} - \xi.$

So the volume change of this fluid parcel is $\frac{\Delta V}{V} = \frac{(A)\,\xi}{(A)(\lambda/2)} = \frac{2\xi}{\lambda}$ — cross-sectional area.

$$\frac{\Delta p}{p} = \gamma \frac{\Delta V}{V} = \gamma \frac{2\xi}{\lambda} \Rightarrow \Delta p = \underline{\left(\frac{2\xi\gamma}{\lambda}\right)p}$$

change in pressure / ambient pressure

THIS = THAT

Um... yeah, there is a point to this... look:

Newton's 2nd law: $F = m\,a$

$$(\Delta p)(A) = \overbrace{\rho A\left(\tfrac{\lambda}{2}\right)}^{m}\left(\frac{\partial^2 \xi}{\partial t^2}\right) \Rightarrow \Delta p = \underline{\left(\rho \frac{\lambda}{2}\right)\left(\frac{\partial^2 \xi}{\partial t^2}\right)} \Longrightarrow p\left(\frac{2\xi\gamma}{\lambda}\right) = \left(\frac{\rho\lambda}{2}\right)\left(\frac{\partial^2 \xi}{\partial t^2}\right)$$

Well, we can simplify a bit, but let's ASSUME THE ACOUSTIC WAVE IS HARMONIC: $\frac{\partial^2 \xi}{\partial t^2} = \omega^2 \xi$ where $\omega^2 = (2\pi f)^2$

$$\Rightarrow p\gamma\frac{2\xi}{\lambda} = 2\rho\lambda\pi^2 f^2 \xi \Rightarrow f^2 = \frac{p\gamma}{\pi^2\lambda^2\rho} \Rightarrow \omega^2 = k^2 c^2 \text{ where } k \equiv \frac{2\pi}{\lambda} \sim \text{the WAVENUMBER.}$$

In an adiabatic fluid (with $f \ll f_0$ from above), sound propagates without dispersion. A non-dispersive acoustic wave propagates at the same speed regardless of frequency.

From Ch 1 (kinetic theory), $c = \sqrt{\frac{\gamma p}{\rho}}$

We know $\Delta p = p\left(\frac{2\xi}{\lambda}\right) \sim \gamma k \xi = \frac{\omega}{c}\xi\gamma \sim \gamma\frac{v}{c}$ where $v \sim$ vibration velocity (particle velocity).
$v \equiv \omega\xi$

The $\underline{\Delta p} = (p\gamma)\frac{v}{c} = (\gamma p)(c)\frac{v}{c^2} = (\gamma p)(c)\left(\frac{v}{1}\right)\left(\frac{\rho}{\gamma p}\right) = \underline{\rho c v} \Rightarrow \Delta p = \rho c v$

"Is v Lagrangian or Eulerian" ~ SEE Westervelt paradox...

$$\Longrightarrow \frac{\Delta p}{v} = \rho c \sim \text{Acoustic Impedance, } Z.$$

Uh, well, remember electric circuits? $\Delta p = Z\, v$

"VOLTAGE" = "RESISTANCE" × "CURRENT"

	ξ	v	a	p
Particle Displ. ξ	—	v/ω	a/ω^2	$p/(\omega \cdot Z)$
Particle Vel. v	$\xi \cdot \omega$ ($\sim 2\pi f$)	—	a/ω	p/Z
Particle Accel. a	$\xi \cdot \omega^2$	$v \cdot \omega$	—	$\frac{p\omega}{Z}$
Sound pressure p	$\xi \cdot \omega \cdot Z$	$v \cdot Z$	$\frac{aZ}{\omega}$	—
Sound intensity I	$\xi^2 \omega^2 Z$	$v^2 Z$	$\frac{a^2 Z}{\omega^2}$	p^2/Z
Sound energy density e	$\xi^2 \cdot \omega^2 \cdot \rho$	$v^2 \rho$	$\frac{a^2 \rho}{\omega^2}$	p^2/Zc
Sound power P_{ac}	$\xi^2 \omega^2 Z A$	$v^2 Z A$	$\frac{a^2 Z A}{\omega^2}$	$\frac{p^2 A}{Z}$

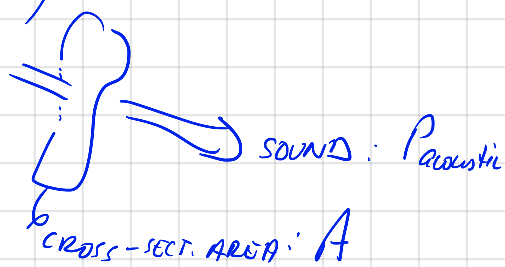
$P_{ac} = I \cdot A$

ALL OF THIS IS FOR A SIMPLE PLANE PROGRESSIVE ACOUSTIC WAVE.

$$\text{Sound intensity} = \frac{P_{acoustic}}{A}$$

$P_{acoustic}$ ~ acoustic power

A ~ cross-sectional area



A little more on acoustics...

ONE-DIM... WAVE EQUATION: $u_{tt} - c^2 u_{xx} = 0$ OR $\xi_{tt} - c^2 \xi_{xx} = 0$

"I want to know more!" ⇒ Fritz John

But the solution to this equation is $u(x,t) = f(x-ct) + g(x+ct)$

If the wave is sinusoidal, then we can write the solution as $u(x,t) = A \cos \omega\left(t - \frac{x}{c}\right)$

A ~ amplitude, ω ~ $2\pi f$, c ~ speed of sound; phase velocity

$$= A \cos\left(\frac{2\pi t}{T} - \frac{x}{\lambda}\right) = A\cos(kx - \omega t)$$

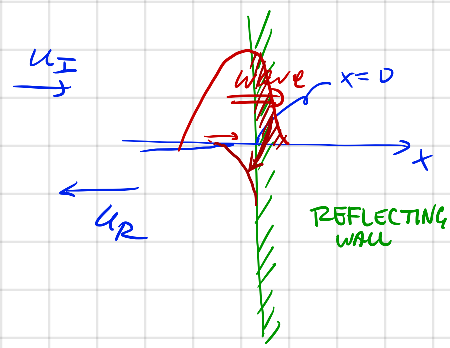
T ~ time period

k ~ wavenumber: $k \equiv \frac{\omega}{c} = \frac{2\pi}{\lambda}$

Wave reflection

NORMAL reflection ⇒

OBLIQUE reflection



INCIDENT:

$$u_I = F_I\left(t - \frac{x}{c}\right) \Rightarrow \overset{\text{REFLECTION}}{u_R = F_R\left(t + \frac{x}{c}\right)}$$

ON THE BOUNDARY, WHAT'S $u = ?$ IT'S ZERO. So $u(\overset{x=0}{x}, t) = u(0,t) = u_I(0,t) + u_R(0,t) \Rightarrow$

$$u(0,t) = F_I(t) + F_R(t) = 0$$

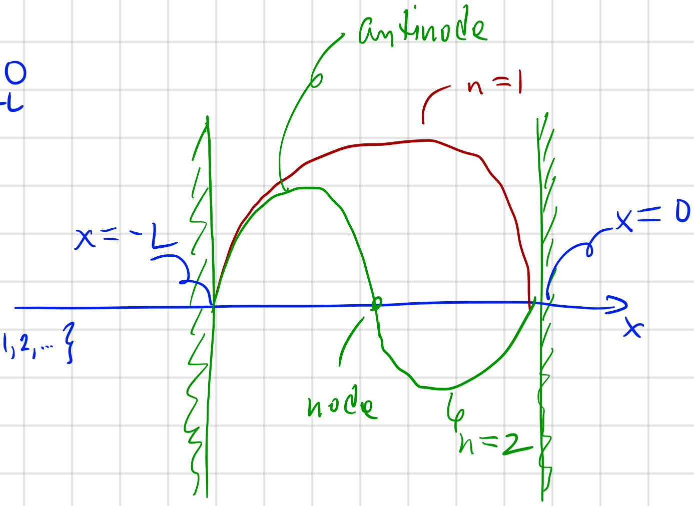
$$\Rightarrow F_R(t) = -F_I(t)$$

$$\Rightarrow u(x,t) = F_I\left(t - \frac{x}{c}\right) + F_R\left(t + \frac{x}{c}\right) = F_I\left(t - \frac{x}{c}\right) - F_I\left(t + \frac{x}{c}\right)$$

If the wave is a harmonic sinusoid, then

$$u(x,t) = A\left[\cos \omega\left(t - \frac{x}{c}\right) - \cos \omega\left(t + \frac{x}{c}\right)\right] = 2A \sin \omega t \sin \frac{\omega x}{c} \sim \text{STANDING WAVE!}$$

Resonator: 2 walls of perfect reflection, one at $x=0$, another at $x=-L$



In this arrangement, we have $u(x,t)\big|_{x=0}=0$ AND $u(x,t)\big|_{x=-L}=0$

We can use our solution from the one-wall case

$$u(x,t)=2A\sin\omega t\,\sin\frac{\omega x}{c}$$

and substitute in $u(x,t)\big|_{x=-L}=0 \Rightarrow \frac{\omega L}{c}=n\pi$ for $n\in\{1,2,\dots\}$

Standing wave ratio: (SWR)

If the reflection off of one of the walls isn't perfect, we'd have a standing wave AND a traveling wave:

$$u(x,t)=A_I\cos\omega\left(t-\frac{x}{c}\right)-A_R\cos\omega\left(t+\frac{x}{c}\right)=$$

$$\underbrace{A_I\cos\omega\left(t-\frac{x}{c}\right)-A_R\cos\omega\left(t-\frac{x}{c}\right)}_{(A_I-A_R)\cos\omega\left(t-\frac{x}{c}\right)}+\underbrace{\left(A_R\cos\omega\left(t-\frac{x}{c}\right)-A_R\cos\omega\left(t+\frac{x}{c}\right)\right)}_{A_R\left[\cos\omega\left(t-\frac{x}{c}\right)-\cos\omega\left(t+\frac{x}{c}\right)\right]}$$

$$(A_I-A_R)\cos\omega\left(t-\frac{x}{c}\right)+(A_I+A_R)\sin\omega t\,\sin\frac{\omega x}{c}$$

$$=(A_I-A_R)\cos\omega\left(t-\frac{x}{c}\right)+(A_I+A_R)\sin\omega t\,\sin\frac{\omega x}{c}$$

DEFINE the standing wave ratio (SWR) as $SWR=\dfrac{A_I+A_R}{A_I-A_R}$: $SWR\to\infty$ for a perfect standing wave

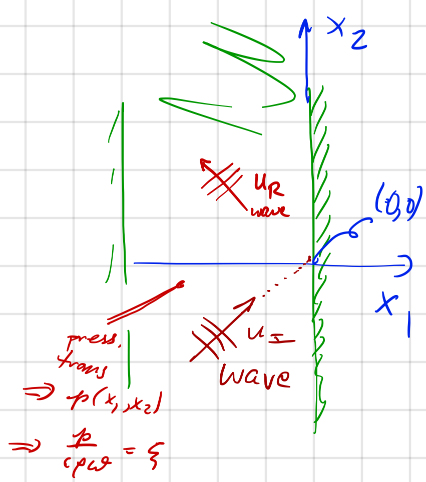
$SWR\to 1$ for a perfect traveling wave. (progressive)

Some books write the reflection coefficient, $r\equiv A_R/A_I \Rightarrow SWR=\dfrac{1+r}{1-r}$.

Oblique waves

INCIDENT WAVE:

$$u_I=A\cos(\omega t-k_1x_1-k_2x_2)$$ ← $\underbrace{\vec{k}}_{\text{wave vector}}=k_1\underbrace{\hat{e}_1}_{\hat{e}_\cdot\sim\text{ unit vector along }(\cdot)}+k_2\hat{e}_2$



REFLECTED WAVE:

$$u_R=-A\cos(\omega t-k_1'x_1-k_2'x_2)$$

NET WAVE: $u=u_I+u_R=2A\sin\left(\omega t-\frac{k_1+k_1'}{2}x_1-\frac{k_2+k_2'}{2}x_2\right)\sin\left(\frac{k_1-k_1'}{2}x_1+\frac{k_2-k_2'}{2}x_2\right)$

BOUNDARY CONDITIONS: $x_1=0$, $u=0 \Rightarrow k_2'=k_2$; AT THE WALL, THROUGH A REFLECTION, $\|\vec{k}\|$ = constant.

The resulting wave motion is $u(x_1, x_2, t) = 2A \sin(\omega t - k_2 x_2) \sin k_1 x_1$.

Phase velocity of wave? $c_\phi = \frac{\omega}{k_2}$

PHASE VELOCITY OF THIS WAVE, $c_\phi \neq c$ ~ SPEED OF A SIMPLE SOUND WAVE IN MEDIA