MAE 207 LECTURE

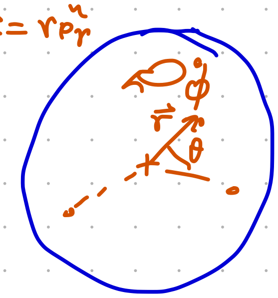
A4 DUE THURSDAY.

$$\vec{F} = \left(-\frac{\alpha}{r^2} + \frac{\beta}{r^3}\right)\hat{e}_r, \quad \vec{F} \sim \text{conservative}: \quad V(\vec{r}) = -\int \vec{F}(\vec{r})\,dr = \int \vec{F}(\vec{r})\,dr\,\hat{e}_r$$

$$\vec{F}_{cb} = -\frac{G m_1 m}{r}\hat{e}_r + \vec{F}$$

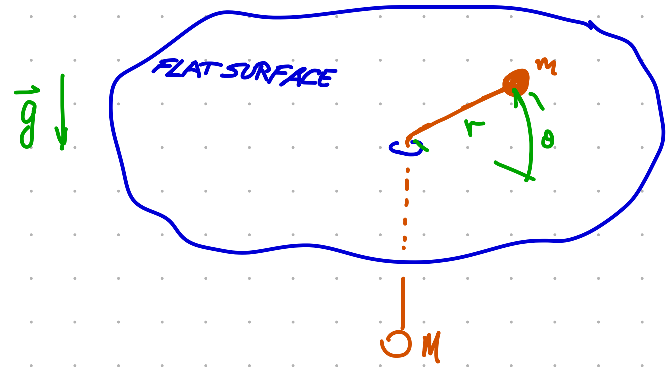
$\vec{r} = r\hat{e}_r$

$\dot{\vec{r}} = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r$



WHIRLYGIG –

- NO STRETCH IN CORD
- NO FRICTION
- CORD LENGTH is L



$$T = \frac{1}{2}M\dot{r}^2 + \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right)$$

$$V = -Mg(L-r)$$

$$L = T - V =$$

$$\frac{1}{2}M\dot{r}^2 + \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + Mg(L-r)$$

2 DOF: r, θ

θ DOESN'T APPEAR explicitly ...

IGNORABLE CDT θ

$$R = c_\theta\dot{\theta} - L$$

const momentum, generalized: $c_\theta = p_\theta = \dfrac{\partial L}{\partial\dot{\theta}} = \dfrac{\partial T}{\partial\dot{\theta}} = mr^2\dot{\theta}$

So $$R = c_\theta\dot{\theta} - \frac{1}{2}M\dot{r}^2 - \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - Mg(L-r)$$

NONSTEADY ROUTHIAN...

$= mr^2\dot{\theta} - \cdots$

$= \frac{1}{2}m\dot{r}^2 - \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 - Mg(L-r)$

For steady motion, THE NONIGNORABLE COORDINATES HAVE NO MOTION $\Rightarrow$ GEN'L VELOCITIES AND MOMENTA ARE ZERO $\Rightarrow$ $\dot{r} = p_r = 0 \Rightarrow \ddot{r} = 0$ (NON IGNORABLE CDT: r)

$$R = c_\theta\dot{\theta} - \frac{1}{2}mr^2\dot{\theta}^2 + Mg(r-L) \leftarrow \text{STEADY ROUTHIAN}$$

LOOK: $c_\theta = mr^2\dot{\theta}$ (from $\frac{\partial L}{\partial\dot{\theta}} = p_\theta = c_\theta$)

$$\Rightarrow \dot{\theta} = \frac{c_\theta}{mr^2} \Rightarrow \dot{\theta}^2 = \frac{c_\theta^2}{m^2r^4}$$

$$\Rightarrow R = \frac{c_\theta^2}{mr^2} - \frac{1}{2}mr^2\frac{c_\theta^2}{m^2r^4} + Mg(r-L)$$

$$= \frac{c_\theta^2}{2mr^2} + Mg(r-L)$$

ALSO: $\frac{\partial R}{\partial q_k} = 0$ for $k \sim$ NON IGNORABLE COORDINATE...

BECAUSE $\cancel{\frac{d}{dt}\left(\frac{\partial R}{\partial \dot{q}_k}\right)} \to 0$ ALREADY (STEADY MOTION!) $- \frac{\partial R}{\partial q_k} = 0 \Rightarrow \frac{\partial R}{\partial q_k} = 0$

$$\frac{\partial R}{\partial r} = 0 = Mg - \frac{c_\theta^2}{mr^3} \Rightarrow Mg = \frac{c_\theta^2}{mr^3}$$

SUBSTITUTE BACK IN FOR c_θ? $c_\theta^2 = m^2r^4\dot{\theta}^2 \Rightarrow$

$$Mg(\cancel{mr^3}) = m^{\cancel{2}}r^{\cancel{4}}\dot{\theta}^2 \Rightarrow Mg = mr\dot{\theta}^2$$

$$r = \frac{Mg}{m\dot{\theta}^2}$$ — for steady motion to occur.

IS IT STABLE? LET'S FIND OUT... BY A SMALL PERTURBATION S_k $\Rightarrow$ $r = r_0 + s$

NEW EXPRESSION — STEADY — PERTURBATION

WHERE $r_0 = \dfrac{Mg}{m\dot{\theta}_0^2}$ as a constant for the original steady motion behavior

THEN $\tilde{R} = c_\theta \dot{\theta} - L =$

WE HAVE TO RELY ON THE NON-STEADY ROUTHIAN, BUT WE CAN STILL USE THE ROUTHIAN INSTEAD OF THE LAGRANGIAN.

$$\frac{c_\theta^2}{2m\cancel{r^2}} - \frac{1}{2}M\cancel{\dot{r}^2} - \frac{1}{2}m\left(\dot{r}^2 + \cancel{r^2\dot{\theta}^2}\right) - Mg(L-r)$$

$(r_0+s)^2$ $\quad (\dot{r}_0 + \dot{s})^2$ $\quad 0$ $\quad (\cancel{\dot{r}_0} + \dot{s})^2$ $\quad 0$

HEY: WE TOOK THIS TERM INTO ACCOUNT: IT IS $\dfrac{c_\theta^2}{2mr^2}$

$$\tilde{R} = \frac{c_\theta^2}{2m(r_0+s)^2} - \frac{1}{2}M\dot{s}^2 - \frac{1}{2}m\dot{s}^2 + Mg(r_0 + s - L)$$

LET'S USE THE ROUTHIAN EQUATION IN OUR PERTURBATION, S

$$\frac{d}{dt}\left(\frac{\partial \tilde{R}}{\partial \dot{s}}\right) - \frac{\partial \tilde{R}}{\partial s} = 0 \qquad (m \cancel{r}^2 \dot{\theta})^2 \Rightarrow m^2 \dot{\theta}^2 (r_0+s)^4$$

$(r_0+s)^2$

$$-(M+m)\ddot{s} - Mg + \frac{c_\theta^2}{m(r_0+s)^3} = 0$$

$$(M+m)\ddot{s} = -Mg + m\dot{\theta}^2(r_0+s) \Rightarrow$$

$$(M+m)\ddot{s} = m\dot{\theta}^2 s \qquad -Mg + m\dot{\theta}^2 r_0 = 0$$ IF WE WANT STEADY MOTION...

HOMOGENEOUS SOLUTION TO THIS $\left((M+m)\ddot{s} - m\dot{\theta}^2 s = 0\right)$ IS

$$s(t) = A \sinh\left(\sqrt{\frac{m\dot{\theta}^2}{(M+m)}}\, t\right) + B\cosh\left(\sqrt{\frac{m\dot{\theta}^2}{M+m}}\, t\right)$$

$$t=0 \Rightarrow s(t=0)=0 \Rightarrow$$

$$s(t) = A\sinh\left(\sqrt{\frac{m\dot{\theta}^2}{M+m}}\, t\right) \longrightarrow$$ HEY: IT'S UNSTABLE, BECAUSE THIS BLOWS UP!

VIRTUAL POWER

- VERY SIMILAR TO VIRTUAL WORK $\left(\delta W = \sum \vec{F}\cdot\delta\vec{r} = 0 = \sum Q\,\delta q = 0\right)$
- FOR holonomic systems, PRODUCES IDENTICAL RESULTS...

– FOR NON-holonomic systems, VIRTUAL POWER IS HELPFUL.

DYNAMICS + VIRTUAL WORK: $\underbrace{\left(\sum_{j=1}^{m} \vec{F}_j - m\ddot{\vec{r}}\right) \cdot \delta\vec{r} = 0}_{\text{D'Lambert's principle}}$

VIRTUAL POWER: $\left(\sum_{j=1}^{m} \vec{F}_j - m\ddot{\vec{r}}\right) \cdot \underbrace{\delta(\dot{\vec{r}})}_{\text{variation in velocities consistent with the constraints ...}} = 0$

$$\left(\sum_{j=1}^{m} \vec{F}_j - m\ddot{\vec{r}}\right) \cdot \frac{\partial \dot{\vec{r}}}{\partial \dot{q}} \delta \dot{q} = 0$$

VIRTUAL POWER METHOD states that for a virtual change in the system's velocities consistent with the system's constraints, the power put into the system $\delta\hat{W}$ by the applied forces less the change in the system's momentum ($m\ddot{\vec{r}}$) is ZERO.

NON HOLONOMIC CONSTRAINTS: $\underbrace{\sum_{i=1}^{n} A_{ij}\dot{q}_i + A_{0j}(t) = 0}_{\text{Pfaffian AGAIN!}}$