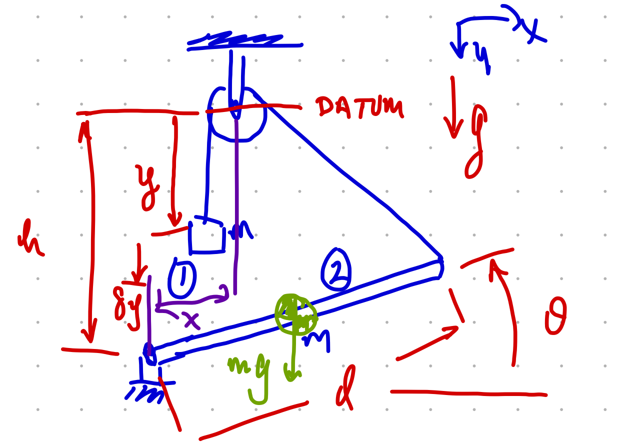


VIRTUAL WORK

ONE DEGREE OF FREEDOM.

VIRTUAL WORK: $\delta W = 0$

$$\delta W = \sum \vec{F}\cdot\delta\vec{r} = \cancel{m\vec{g}\cdot\delta\vec{r}_1}^{\,mg\,\delta y} + m\vec{g}\cdot\delta\vec{r}_2$$

$\vec{r}_2 = (\ )\hat{e}_x + (-\frac{1}{2}l\sin\theta)\hat{e}_y$

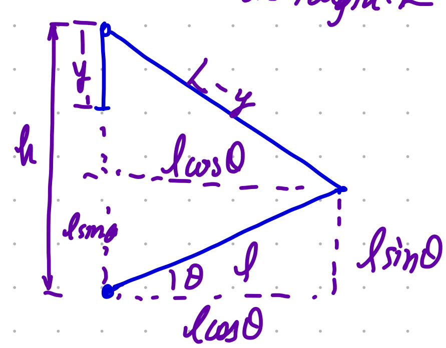
$\Rightarrow \delta\vec{r}_2 = -\frac{1}{2}l\cos\theta\,\delta\theta\,\hat{e}_y + (\dots)\hat{e}_x$

$$= mg\,\delta y + mg\left(\frac{l}{2}\right)\cos\theta\,\delta\theta\,(-1)$$

$$= 0 = mg\,\delta y - \frac{1}{2}mgl\cos\theta\,\delta\theta$$

THE BEAM IS PINNED AT LEFT DIRECTLY UNDER THE PULLEY: $x = 0$

cable length: L



$$(L-y)^2 = l^2\cos^2\theta + (h - l\sin\theta)^2$$

$$\Rightarrow L - y = \sqrt{l^2\cos^2\theta + l^2\sin^2\theta + h^2 - 2hl\sin\theta}$$

$$= \sqrt{l^2 + h^2 - 2hl\sin\theta}$$

$$y = L - \sqrt{l^2 + h^2 - 2hl\sin\theta}$$

$$\delta y = 0 + \frac{1}{\cancel{2}}\left(l^2 + h^2 - 2hl\sin\theta\right)^{-1/2}\left(+\cancel{2}hl\cos\theta\right)\underline{\underline{\delta\theta}}$$

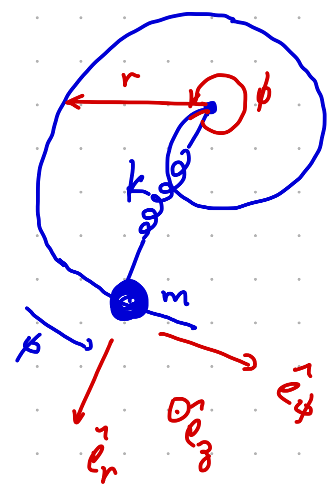
$$0 = mg\frac{hl\cos\theta}{\sqrt{l^2+h^2-2hl\sin\theta}}\,\delta\theta - \frac{1}{2}mgl\cos\theta\,\delta\theta .$$

$$\theta = \cos^{-1}\left(\frac{l^2-15h^2}{2hl}\right)$$

LAGRANGE'S EQUATIONS EXAMPLE:

"WHIRLYGIG"

- $\phi = 2\pi r \Rightarrow \underline{\dot{\phi} = 2\pi\dot{r}}$
- UNSTRETCHED LENGTH OF SPRING IS ZERO.



① POSITION VECTOR $\vec{r} = r\hat{e}_r$

② VELOCITY VECTOR

$$\dot{\vec{r}} = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r$$

$$= \dot{r}\hat{e}_r + r(\underbrace{\vec{\omega}}_{\dot{\phi}\hat{e}_z}\times\hat{e}_r) = \dot{r}\hat{e}_r + r\dot{\phi}\,\underbrace{\hat{e}_z\times\hat{e}_r}_{\hat{e}_\phi}$$

$$= \dot{r}\hat{e}_r + r\underbrace{\dot{\phi}}_{2\pi\dot{r}}\hat{e}_\phi = \dot{r}\hat{e}_r + 2\pi r\dot{r}\hat{e}_\phi$$

③ KINETIC ENERGY:

$$T = \frac{1}{2}m\dot{\vec{r}}\cdot\dot{\vec{r}}$$

$$= \frac{1}{2}m\left(\dot{r}^2 + 4\pi^2 r^2\dot{r}^2\right)$$

$$= \frac{1}{2}m\dot{r}^2\left(1 + 4\pi^2 r^2\right)$$

④ POTENTIAL ENERGY:

$$V = \frac{1}{2}k(\vec{r}\cdot\vec{r})$$

$$= \frac{1}{2}kr^2$$

⑤ Lagrangian: $$\mathcal{L} = T - V = \frac{1}{2}m\dot{r}^2(1+4\pi^2r^2) - \frac{1}{2}kr^2$$

⑥ Equation of motion: r

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{r}}\right) - \frac{\partial\mathcal{L}}{\partial r} = \cancel{Q_r}^{\,0}$$

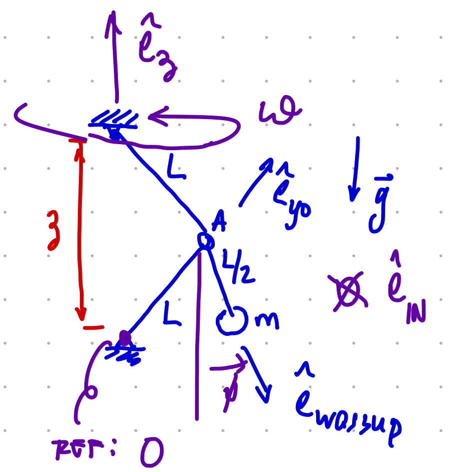
$$\frac{\partial \mathcal{L}}{\partial \dot r} = m\dot r\left(1+4\pi^2 r^2\right) \Rightarrow m\ddot r\left(1+4\pi^2 r^2\right) + m\dot r\left(4\pi^2 r \dot r\right)(2)$$

$$\frac{\partial \mathcal{L}}{\partial r} = m\dot r^2\left(4\pi^2 r\right) - kr$$

$$m\ddot r\left(\left(1+4\pi^2 r^2\right) + 8m\pi^2 \dot r^2 r - 4\pi^2 m \dot r^2 r + kr = 0$$

$$m\ddot r\left(1+4\pi^2 r^2\right) + 4m\pi^2 \dot r^2 r + kr = 0$$

3D KINEMATICS



$$\vec r_{OA} = L\,\hat e_{y0}$$

$$\vec r_{OM} = \vec r_{OA} + \frac{L}{2}\hat e_{wassup}$$
$$= L\hat e_{y0} + \frac{L}{2}\hat e_{wassup}$$

$$\vec\omega' = \vec\omega + (-\dot\phi)\hat e_{IN}$$
$$= \omega \hat e_z - \dot\phi\, \hat e_{IN}$$

$$\dot{\vec r}_{OM} = L\underbrace{\dot{\hat e}_{y0}}_{\vec\omega \times \hat e_{y0}} + \frac{L}{2}\underbrace{\dot{\hat e}_{wassup}}_{\vec\omega' \times \hat e_{wassup}}$$

$$\omega \hat e_z \times \hat e_{y0} = \omega \hat e_{IN}$$

NEEDS SIN OF ANGLE BETWEEN THEM...

NO

$$\boxed{\begin{aligned} &= L\omega\, \hat e_{IN} + \frac{L}{2}\left(\omega \hat e_z - \dot\phi\, \hat e_{IN}\right) \times \hat e_{wassup} \\ &= L\omega\, \hat e_{IN} + \frac{L}{2}\omega\, \hat e_{IN} - \frac{L}{2}\dot\phi\, \hat e_{IN} \times \hat e_{wassup}\end{aligned}}$$

IN

this way

wassup