3 - Deformable bodies: elasticity: $\vec{x}' = \vec{F} \cdot \vec{x}$ (second order tensor)

7 - Piezo (mechanical – electrical; anisotropy; vibrations / acoustics)

7 – Electromechanics

1 - Stability theory, & control (Lyapunov exponents)

7 - Nonlinear phenomena — Strogatz, Steven

5 – Celestial mechanics

1 – Fracture mechanics

1 – Advanced vibrations

Continuous systems, continued...

$$\rho A \ddot{v} + \frac{\partial}{\partial x}\left[\kappa A G (\alpha - v') \right] = p(x,t)$$

$$\rho I \ddot{\alpha} - \frac{\partial}{\partial x}\left[E I \alpha' \right] + \kappa A G (\alpha - v') = 0.$$

BOUNDARY CONDITIONS (at $x=0$, or $x=L$)

(1) $EI\alpha' \delta\alpha = 0 \Rightarrow$ EITHER $\delta\alpha = 0 \Rightarrow \alpha = \alpha^*$ (constant) OR $EI\alpha' = 0$

(2) $\kappa A G (v' - \alpha) \delta v = 0 \Rightarrow$ EITHER $\delta v = 0 \Rightarrow v = v^*$ (constant) OR $\kappa A G (v' - \alpha) = 0$

How to use Hamilton's Principle:

(1) Find all energy (KE, PE, strain E, etc.)

(2) Find the virtual work expression for non-conservative forces.

(3) Subs. into Hamilton's principle...

(4) Integration by parts, find variations of generalized variables.

(5) Check your work

(6) " "

(7) " "

(8) Collect on $\delta(\cdots)$ in and out of integrals

(9) EOMs of motion & BCs

(10) Profit.

BEAM EQUATIONS: $\frac{\partial}{\partial x}\left[\kappa A G(\alpha - v')\right] = p(x,t) - \rho A \ddot{v} \ldots$

IF κ, A, G ARE CONSTANT, THEN

$$\kappa A G(\alpha' - v'') = p(x,t) - \rho A \ddot{v} \Rightarrow$$

$$\alpha' = v'' + \frac{1}{\kappa A G}\left[p(x,t) - \rho A \ddot{v}\right]$$

THE other EQUATION WAS

$$\rho I \ddot{\alpha} - \frac{\partial}{\partial x}\left[EI\alpha'\right] + \kappa A G(\alpha - v') = 0 \Rightarrow$$

$$\Rightarrow \rho I \ddot{\alpha}' - \frac{\partial^2}{\partial x^2}\left[EI\alpha'\right] + \kappa A G(\alpha' - v'') = 0$$

$$\rho I \frac{\partial^2}{\partial t^2}\left[v'' + \frac{1}{\kappa A G}(p - \rho A \ddot{v})\right] -$$

$$\frac{\partial^2}{\partial x^2}\left[EI\left(v'' + \frac{1}{\kappa A G}(p - \rho A \ddot{v})\right)\right] + (p(x,t) - \rho A \ddot{v}) = 0$$

If E, I are constant,

$$EI\frac{\partial^4 v}{\partial x^4} - \frac{\partial^2}{\partial x^2}\left(\frac{EI}{\kappa A G}(p - \rho A \ddot{v})\right) + (p - \rho A \ddot{v})$$

$$+ \rho I \frac{\partial^4 v}{\partial x^2 \partial t^2} + \frac{\rho I}{\kappa A G}\left(\frac{\partial^2 p}{\partial t^2} - \rho A \frac{\partial^4 v}{\partial t^4}\right) = 0$$

(4)

(1) $EI\, v^{IV} + p - \rho A \ddot{v} = 0$ ~ EULER-BERNOULLI BEAM (THIN BEAM)

(2) ROTARY INERTIA (ROTATORY IN BRITISH ENGLISH)

(3) SHEAR DEFORM.

(4) COUPLING: SHEAR & ROTARY INERTIA.

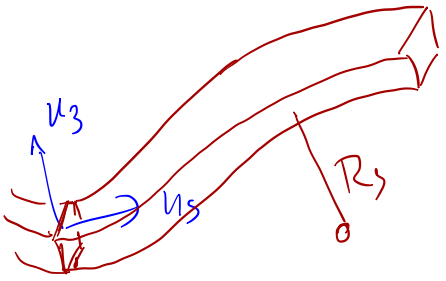
ARCH (THIN ARCH: NO SHEAR DEFM, NO ROTARY INERTIA)

$$\textcircled{1}\quad EI\left[\frac{\partial^3}{\partial s^3}\left(\frac{u_s}{R_s}\right)-\frac{\partial^4 u_3}{\partial s^4}\right]-\frac{EA}{R_s}\left[\frac{\partial u_s}{\partial s}+\frac{u_s}{R_s}\right]+p_3'=\rho A\frac{\partial^2 u_s}{\partial t^2}$$

$$\textcircled{2}\quad \frac{EI}{R_s}\left[\frac{\partial^2}{\partial s^2}\left(\frac{u_s}{R_s}\right)-\frac{\partial^3 u_3}{\partial s^3}\right]+EA\left[\frac{\partial^2 u_s}{\partial s^2}+\frac{\partial}{\partial s}\left(\frac{u_3}{R_s}\right)\right]+p_s'=\rho A\frac{\partial^2 u_3}{\partial t^2}$$

$s \sim$ WORD. ALONG THE ARCH

$R_s \sim$ RADIUS OF CURVATURE AS A FUNCTION OF POSITION (s) ALONG ARCH

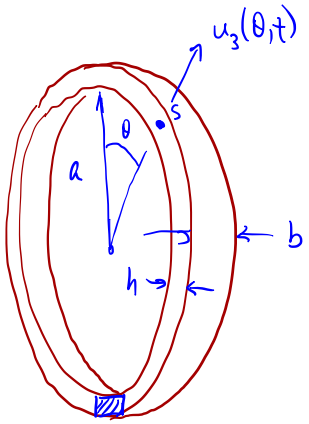


$u_3(s,t) \sim$ TRANSVERSE DISP'

$u_s(s,t) \sim$ AXIAL DISP.

$p_3(s,t) \sim$ force / unit length (transverse)

$p_s(s,t) \sim$ " " (axial)



CIRCULAR RING: $R_s = a \sim$ RADIUS OF THE RING.

$s = a\theta$

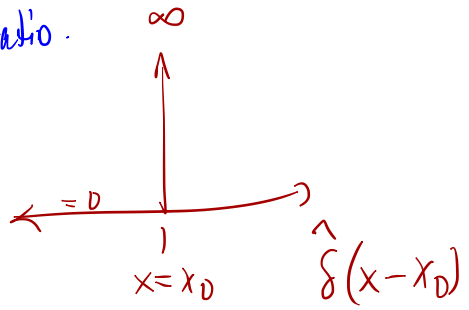
(BENDING STIFFNESS)

$$\frac{D}{a^4}\left(\frac{\partial^4 u_3}{\partial\theta^4}+2\frac{\partial^2 u_3}{\partial\theta^2}+u_3\right)+\rho h\frac{\partial^2 u_3}{\partial t^2}=f_0\hat{\delta}(\theta-\theta_0)$$

BENDING STIFFNESS: $D = \dfrac{E t^3}{12(1-\nu^2)}$

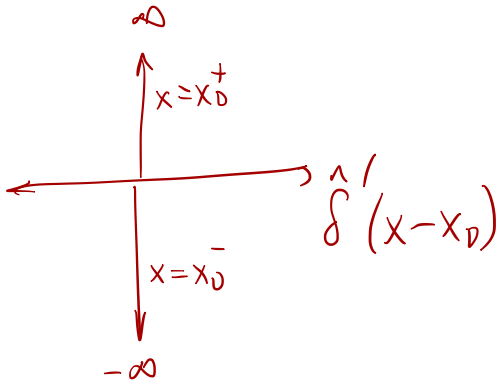
ν — Poisson's ratio.

DIRAC DELTA DISTRIBUTION (NOT FUNCTION)



$$\delta \int_{t_1}^{t_2} \iiint_{\forall} f\, \hat{\delta}(\vec{r} - r_0)\, d\forall\, dt = 0$$ — for appl' force

$$\delta \int_{t_1}^{t_2} \iiint_{\forall} m\, \hat{\delta}'(r - r_0)\, d\forall\, dt = 0$$ — for appl' moment



See W. SOEDEL, VIBRATIONS OF PLATES AND SHELLS 3RD EDITION (BUT NOTE LH COORD SYS, MANY MISTAKES!)

FLEXIBLE BODIES W/ DYNAMIC MOTION: BARUH, HAIM: Analytical Dynamics McGraw-Hill 007-365990 §11.6-10

Dynamics of Railway Bridges: Frýbá, L 0727720449

Vibr. of Solids and Structures " " 0727728419